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TOP QUARK MASS PREDICTIONS FROM GAUGE-YUKAWA UNIFICATION [†]

Dedicated to the memory of our very good friend Roger Decker

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Abstract

Gauge-Yukawa Unification (GYU) is a renormalization group invariant functional relation among gauge and Yukawa couplings which holds beyond the unification point in Grand Unified Theories (GUTs). Here, GYU is obtained by requiring finiteness and reduction of couplings to all orders in perturbation theory. We examine the consequences of GYU in various supersymmetric GUTs and in particular the predictions for the top quark mass. These predictions are such that they distinguish already GYU from ordinary GUTs. Moreover, when more accurate measurements of the top quark mass are available, it is expected that it will be possible to discriminate among the various GYUs.

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1 Introduction

The apparent success of the standard model (SM) in describing the elementary particles and their interactions is spoiled by the presence of the plethora of free parameters. Therefore, it provides a challenge to theorists to attempt to understand at least some of these parameters.

The traditional way to reduce the independent parameters of a theory is the introduction of a symmetry. Grand Unified Theories (GUTs) [1, 2, 3] are representative examples of such attempts, and some of them are certainly successful to some extent. For instance, the minimal $SU(5)$ reduces by one the gauge couplings of the SM and gives us a testable prediction for one of them. In fact, LEP data [4] seem to suggest that a further symmetry, namely $N = 1$ global supersymmetry [5] should also be required to make the prediction viable. Furthermore, GUTs also relate Yukawa couplings among themselves, which in turn might lead to testable predictions for the parameters of the SM. The prediction of the ratio m_τ/m_b [6] in the minimal $SU(5)$ was an example of a successful reduction of the independent parameters of this sector of the theory. On the other hand, requiring more symmetry (*e.g.* $SO(10)$, E_6 , E_7 , E_8) does not necessarily lead to more predictions for the SM parameters, due to the presence of new degrees of freedom, various ways and channels of breaking the theory, etc. An extreme case from this point of view are superstrings, which have huge symmetries, but no predictions for the SM parameters.

In a series of papers [7, 8, 9, 10] we have proposed that a natural gradual extension of the GUTs ideas, which preserves their successes and enhances the predictions, is to attempt to relate the gauge and Yukawa couplings of a GUT, or in other words, to achieve Gauge-Yukawa Unification (GYU).

Searching for a symmetry that could provide such a unification, one is led to introduce a symmetry that relates fields with different spins, *i.e.* supersymmetry, and in particular $N = 2$ supersymmetry [11]. Unfortunately $N = 2$ supersymmetric theories have serious phenomenological problems due to light mirror fermions. We expect that a GYU is a functional relationship which is derived by some principle. In superstring theories or in composite models there exist relations among gauge and Yukawa couplings, although in

practice both kinds of theories have more problems than the SM.

Before turning to our attempts, let us mention some earlier ones in order to make clear which are really the predictions in each case. Veltman, Decker and Pestieau [12], by requiring the *absence of quadratic divergences* in the SM, found that the following relationship has to hold:

$$\begin{aligned} m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2 \end{aligned} \quad (1)$$

A very similar relation is obtained demanding spontaneous breaking of supersymmetry via F-terms. In that case one obtains [13]

$$\sum_j (-1)^{2j} (2j+1) m_j^2 = 0, \quad (2)$$

where j is the spin of the particle. When this formula is applied to the SM we obtain a relation which differs from Eq.(1) only in the coefficient of m_H^2 . In both cases a prediction for the top quark was possible only when it was permitted experimentally to neglect the m_H as compared to $m_{W,Z}$ with the result $m_t = 69$ GeV. Otherwise there is only a quadratic relation among m_t and m_H .

A celebrated relation among gauge and Yukawa couplings is the Pendleton-Ross (P-R) *infrared* fixed point [14]. The P-R proposal, involving the Yukawa coupling of the top quark g_t and the strong gauge coupling α_3 , was that the ratio α_t/α_3 , where $\alpha_t = g_t^2/4\pi$, has an infrared fixed point. This assumption predicted $m_t \sim 100$ GeV, and therefore it is ruled out. Moreover, it has been shown [21] that the P-R conjecture is not justified at two-loops. On the contrary, the ratio α_t/α_3 diverges in the infrared.

Another interesting conjecture, made by Hill [15], is that α_t itself develops an infrared fixed point, leading to the prediction $m_t \sim 280$ GeV.

The P-R and Hill conjectures have been done in the framework on the SM. The same conjectures within the minimal supersymmetric SM (MSSM) lead to the following relations:

$$m_t \simeq 140 \text{ GeV } \sin \beta \quad (P-R) \quad (3)$$

$$m_t \simeq 200 \text{ GeV } \sin \beta \quad (Hill) \quad (4)$$

where $\tan \beta = v_u/v_d$ is the ratio of the two VEV of the Higgs fields of the MSSM. We should stress that in this case there is no prediction for m_t , given that $\sin \beta$ is not fixed from other considerations. Therefore, the conclusion is that all the attempts that have been made so far to extract predictions by some principle, leading to relations among gauge and Yukawa couplings were found either wrong or not predictive enough.

In the following we would like to emphasize an alternative way to achieve unification of couplings, which is based on the fact that within the framework of a renormalizable field theory, one can find renormalization group invariant (RGI) relations among parameters, that can improve the calculability and the predictive power of a theory. In our recent studies [7, 8, 9, 10], we have considered the GYU which is based on the principles of reduction of couplings [16, 17, 18, 19, 20] and finiteness [7, 22, 23, 24, 25, 26, 27, 28]. These principles, which are formulated in perturbation theory, are not explicit symmetry principles, although they might imply symmetries. The former principle is based on the existence of RGI relations among couplings, which preserve perturbative renormalizability. Similarly, the latter one is based on the fact that it is possible to find RGI relations among couplings that keep finiteness in perturbation theory, even to all orders. Applying these principles one can relate the gauge and Yukawa couplings without introducing necessarily a symmetry, nevertheless improving the predictive power of a model.

2 Unification of Couplings by the RGI Method

Let us next briefly outline the idea. Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \dots, g_A) = \text{const.}$, which has to satisfy the partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla} \cdot \vec{\beta} = \sum_{a=1}^A \beta_a \frac{\partial \Phi}{\partial g_a} = 0, \quad (5)$$

where β_a is the β -function of g_a . This PDE is equivalent to the set to ordinary differential equations, the so-called reduction equations (REs) [17],

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \dots, A, \quad (6)$$

where g and β_g are the primary coupling and its β -function, and a does not include it. Since maximally $(A - 1)$ independent RGI “constraints” in the A -dimensional space of couplings can be imposed by Φ_a ’s, one could in principle express all the couplings in terms of a single coupling g . The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_{n=0} \rho_a^{(n+1)} g^{2n+1} , \quad (7)$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [17]. To illustrate this, let us assume that the β -functions have the form

$$\begin{aligned} \beta_a &= \frac{1}{16\pi^2} \left[\sum_{b,c,d \neq g} \beta_a^{(1)bcd} g_b g_c g_d + \sum_{b \neq g} \beta_a^{(1)b} g_b g^2 \right] + \cdots , \\ \beta_g &= \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots , \end{aligned} \quad (8)$$

where \cdots stands for higher order terms, and $\beta_a^{(1)bcd}$ ’s are symmetric in b, c, d . We then assume that the $\rho_a^{(n)}$ ’s with $n \leq r$ have been uniquely determined. To obtain $\rho_a^{(r+1)}$ ’s, we insert the power series (7) into the REs (6) and collect terms of $O(g^{2r+3})$ and find

$$\sum_{d \neq g} M(r)_a^d \rho_d^{(r+1)} = \text{lower order quantities} ,$$

where the r.h.s. is known by assumption, and

$$M(r)_a^d = 3 \sum_{b,c \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} + \beta_a^{(1)d} - (2r+1) \beta_g^{(1)} \delta_a^d , \quad (9)$$

$$0 = \sum_{b,c,d \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} \rho_d^{(1)} + \sum_{d \neq g} \beta_a^{(1)d} \rho_d^{(1)} - \beta_g^{(1)} \rho_a^{(1)} . \quad (10)$$

Therefore, the $\rho_a^{(n)}$ ’s for all $n > 1$ for a given set of $\rho_a^{(1)}$ ’s can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geq 0$.

The possibility of coupling unification described above is without any doubt attractive because the “completely reduced” theory contains only one independent coupling, but it can be unrealistic. Therefore, one often would like to impose fewer RGI constraints, and this is the idea of partial reduction [18].

In the following chapters we would like to consider three different GYU models based on supersymmetric unified theories. Before doing so let us recall here some features

which are common in all cases. Let us consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with the gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} C_{ijk} \phi_i \phi_j \phi_k , \quad (11)$$

where m_{ij} and C_{ijk} are gauge invariant tensors and the matter field ϕ_i transforms according to the irreducible representation R_i of the gauge group G . The renormalization constants associated with the superpotential (11), assuming that supersymmetry is preserved, are

$$\phi_i^0 = (Z_i^j)^{(1/2)} \phi_j , \quad (12)$$

$$m_{ij}^0 = Z_{ij}^{i'j'} m_{i'j'} , \quad (13)$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'} C_{i'j'k'} . \quad (14)$$

The $N = 1$ non-renormalization theorem [30] ensures that there are no mass and cubic-interaction-term infinities and therefore

$$\begin{aligned} Z_{ijk}^{i'j'k'} Z_{i'}^{1/2 i''} Z_{j'}^{1/2 j''} Z_{k'}^{1/2 k''} &= \delta_{(i}^{i''} \delta_j^{j''} \delta_{k)}^{k''} , \\ Z_{ij}^{i'j'} Z_{i'}^{1/2 i''} Z_{j'}^{1/2 j''} &= \delta_{(i}^{i''} \delta_{j)}^{j''} . \end{aligned} \quad (15)$$

As a result the only surviving possible infinities are the wave-function renormalization constants Z_i^j , i.e., one infinity for each field. The one -loop β -function of the gauge coupling g is given by [22]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2} [\sum_i l(R_i) - 3 C_2(G)] , \quad (16)$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . The β -functions of C_{ijk} , by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_{ij} of the matter fields ϕ_i as:

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl} \gamma_k^l + C_{ikl} \gamma_j^l + C_{jkl} \gamma_i^l . \quad (17)$$

At one-loop level γ_{ij} is [22]

$$\gamma_{ij}^{(1)} = \frac{1}{32\pi^2} [C^{ikl} C_{jkl} - 2 g^2 C_2(R_i) \delta_{ij}] , \quad (18)$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i , and $C^{ijk} = C_{ijk}^*$. Since dimensional coupling parameters such as masses and couplings of cubic scalar field terms do not influence the asymptotic properties of a theory on which we are interested here, it is sufficient to take into account only the dimensionless supersymmetric couplings such as g and C_{ijk} . So we neglect the existence of dimensional parameters, and assume furthermore that C_{ijk} are real so that C_{ijk}^2 always are positive numbers. For our purposes, it is convenient to work with the square of the couplings and to arrange C_{ijk} in such a way that they are covered by a single index i ($i = 1, \dots, n$):

$$\alpha = \frac{|g|^2}{4\pi}, \quad \alpha_i = \frac{|g_i|^2}{4\pi}. \quad (19)$$

The evolution equations of α 's in perturbation theory then take the form

$$\begin{aligned} \frac{d\alpha}{dt} &= \beta = -\beta^{(1)}\alpha^2 + \dots, \\ \frac{d\alpha_i}{dt} &= \beta_i = -\beta_i^{(1)}\alpha_i\alpha + \sum_{j,k} \beta_{i,jk}^{(1)}\alpha_j\alpha_k + \dots, \end{aligned} \quad (20)$$

where \dots denotes the contributions from higher orders, and $\beta_{i,jk}^{(1)} = \beta_{i,kj}^{(1)}$.

Given the set of the evolution equations (20), we investigate the asymptotic properties, as follows. First we define [16, 17]

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha}, \quad i = 1, \dots, n, \quad (21)$$

and derive from Eq. (20)

$$\begin{aligned} \alpha \frac{d\tilde{\alpha}_i}{d\alpha} &= -\tilde{\alpha}_i + \frac{\beta_i}{\beta} = \left(-1 + \frac{\beta_i^{(1)}}{\beta^{(1)}}\right)\tilde{\alpha}_i \\ &\quad - \sum_{j,k} \frac{\beta_{i,jk}^{(1)}}{\beta^{(1)}}\tilde{\alpha}_j\tilde{\alpha}_k + \sum_{r=2} \left(\frac{\alpha}{\pi}\right)^{r-1} \tilde{\beta}_i^{(r)}(\tilde{\alpha}), \end{aligned} \quad (22)$$

where $\tilde{\beta}_i^{(r)}(\tilde{\alpha})$ ($r = 2, \dots$) are power series of $\tilde{\alpha}$'s and can be computed from the r -th loop β -functions. Next we search for fixed points ρ_i of Eq. (21) at $\alpha = 0$. To this end, we have to solve

$$\left(-1 + \frac{\beta_i^{(1)}}{\beta^{(1)}}\right)\rho_i - \sum_{j,k} \frac{\beta_{i,jk}^{(1)}}{\beta^{(1)}}\rho_j\rho_k = 0, \quad (23)$$

and assume that the fixed points have the form

$$\rho_i = 0 \text{ for } i = 1, \dots, n' ; \rho_i > 0 \text{ for } i = n' + 1, \dots, n . \quad (24)$$

We then regard $\tilde{\alpha}_i$ with $i \leq n'$ as small perturbations to the undisturbed system which is defined by setting $\tilde{\alpha}_i$ with $i \leq n'$ equal to zero. As we have seen, it is possible to verify at the one-loop level [17] the existence of the unique power series solution

$$\tilde{\alpha}_i = \rho_i + \sum_{r=2} \rho_i^{(r)} \alpha^{r-1} , \quad i = n' + 1, \dots, n \quad (25)$$

of the reduction equations (22) to all orders in the undisturbed system. These are RGI relations among couplings and keep formally perturbative renormalizability of the undisturbed system. So in the undisturbed system there is only *one independent* coupling, the primary coupling α .

The small perturbations caused by nonvanishing $\tilde{\alpha}_i$ with $i \leq n'$ enter in such a way that the reduced couplings, i.e., $\tilde{\alpha}_i$ with $i > n'$, become functions not only of α but also of $\tilde{\alpha}_i$ with $i \leq n'$. It turned out that, to investigate such partially reduced systems, it is most convenient to work with the partial differential equations

$$\left\{ \tilde{\beta} \frac{\partial}{\partial \alpha} + \sum_{a=1}^{n'} \tilde{\beta}_a \frac{\partial}{\partial \tilde{\alpha}_a} \right\} \tilde{\alpha}_i(\alpha, \tilde{\alpha}) = \tilde{\beta}_i(\alpha, \tilde{\alpha}) ,$$

$$\tilde{\beta}_{i(a)} = \frac{\beta_{i(a)}}{\alpha^2} - \frac{\beta}{\alpha^2} \tilde{\alpha}_{i(a)} , \quad \tilde{\beta} \equiv \frac{\beta}{\alpha} , \quad (26)$$

which are equivalent to the reduction equations (22), where we let a, b run from 1 to n' and i, j from $n' + 1$ to n in order to avoid confusion. We then look for solutions of the form

$$\tilde{\alpha}_i = \rho_i + \sum_{r=2} \left(\frac{\alpha}{\pi} \right)^{r-1} f_i^{(r)}(\tilde{\alpha}_a) , \quad i = n' + 1, \dots, n , \quad (27)$$

where $f_i^{(r)}(\tilde{\alpha}_a)$ are supposed to be power series of $\tilde{\alpha}_a$. This particular type of solution can be motivated by requiring that in the limit of vanishing perturbations we obtain the undisturbed solutions (25) [19, 29]. Again it is possible to obtain the sufficient conditions for the uniqueness of $f_i^{(r)}$ in terms of the lowest order coefficients.

3 Finite Unified Model Based on $SU(5)$

According to the discussion in the previous chapter, the non-renormalization theorem ensures that there are no extra mass and cubic-interaction-term renormalizations, implying that the β -functions of C_{ijk} can be expressed as linear combinations of the anomalous dimension matrix γ_{ij} of ϕ^i . Therefore, all the one-loop β -functions of the theory vanish if

$$\beta_g^{(1)} = 0 \text{ and } \gamma_{ij}^{(1)} = 0 \quad (28)$$

are satisfied, where $\beta_g^{(1)}$ and $\gamma_{ij}^{(1)}$ are given in Eqs. (16) and (18) respectively. A very interesting result is that these conditions (28) are necessary and sufficient for finiteness at the two-loop level [22].

A natural question is what happens in higher loops. Since the finiteness conditions impose relations among couplings, they have to be consistent with the REs (6) (this should be so even for the one-loop finiteness). Interestingly, there exists a powerful theorem [27] which provides the necessary and sufficient conditions for finiteness to all loops. The theorem makes heavy use of the non-renormalization property of the supercurrent anomaly [28]. In fact, the finiteness theorem can be formulated in terms of one-loop quantities, and it states that for supersymmetry gauge theories we are considering here, the necessary and sufficient conditions for β_g and β_{ijk} to vanish to all orders are [27]:

- (a) The validity of the one-loop finiteness conditions, i.e., Eq. (28) is satisfied.
- (b) The REs (2) admit a unique power series solution, i.e., the corresponding matrix M defined in Eq. (9) with $\beta_g^{(1)} = 0$ has to be non-singular.

The latter condition is equivalent to the requirement that the one-loop solutions $\rho_a^{(1)}$'s are isolated and non-degenerate. Then each of these solutions can be extended, by a recursion formula, to a formal power series in g giving a theory which depends on a single coupling g , and has β -functions vanishing to all orders.

From the classification of theories with $\beta_g^{(1)} = 0$ [23], one can see that using $SU(5)$ as gauge group there exist only two candidate models which can accommodate three fermion generations. These models contain the chiral supermultiplets **5**, $\bar{\mathbf{5}}$, **10**, $\bar{\mathbf{5}}$, **24** with the multiplicities (6, 9, 4, 1, 0) and (4, 7, 3, 0, 1), respectively. Only the second one contains a **24**-plet which can be used for spontaneous symmetry breaking (SSB) of $SU(5)$ down to

$SU(3) \times SU(2) \times U(1)$. (For the first model one has to incorporate another way, such as the Wilson flux breaking to achieve the desired SSB of $SU(5)$.) Here we would like to concentrate only on the second model.

The most general $SU(5)$ invariant, cubic superpotential of the (second) model is:

$$\begin{aligned} W = & H_a [f_{ab} \overline{H}_b \mathbf{24} + h_{ia} \overline{\mathbf{5}}_i \mathbf{24} + \overline{g}_{ija} \mathbf{10}_i \overline{\mathbf{5}}_j] + p (\mathbf{24})^3 \\ & + \frac{1}{2} \mathbf{10}_i [g_{ija} \mathbf{10}_j H_a + \hat{g}_{iab} \overline{H}_a \overline{H}_b + g'_{ijk} \overline{\mathbf{5}}_j \overline{\mathbf{5}}_k] , \end{aligned} \quad (29)$$

where $i, j, k = 1, 2, 3$ and $a, b = 1, \dots, 4$, and we sum over all indices in W (the $SU(5)$ indices are suppressed). The $\mathbf{10}_i$'s and $\overline{\mathbf{5}}_i$'s are the usual three generations, and the four $(\mathbf{5} + \overline{\mathbf{5}})$ Higgses are denoted by H_a , \overline{H}_a .

Given the superpotential, the $\gamma^{(1)}$'s can be easily computed ($\beta_g^{(1)}$ vanishes of course). To ensure finiteness of the model to all orders, we have to find $\rho^{(1)}$'s that are isolated and non-degenerate solutions of Eq. (6) and are consistent with the vanishing $\gamma^{(1)}$'s. In most of the previous studies of the present model [24, 25], however, no attempt was made to find isolated and non-degenerate solutions, but rather the opposite. They have used the freedom offered by the degeneracy in order to make specific ansätze that could lead to phenomenologically acceptable predictions. Here we concentrate on finding an isolated and non-degenerate solution that is phenomenologically interesting. As a first approximation to the Yukawa matrices, a diagonal solution, that is, without intergenerational mixing, may be considered. It has turned out that this can be achieved by imposing the $Z_7 \times Z_3$ discrete symmetry and a multiplicative Q -parity on W , and that, in order to respect these symmetries, only g_{iii} , \overline{g}_{iii} , f_{ii} and p are allowed to be non-vanishing. Moreover, we have found that under this situation there exists a unique reduction solution that satisfies the finiteness conditions (a) and (b) [7]:

$$\begin{aligned} \alpha_{iii} &= \frac{8}{5} \alpha_{GUT} + O(\alpha_{GUT}^2) , \quad \overline{\alpha}_{iii} = \frac{6}{5} \alpha_{GUT} + O(\alpha_{GUT}^2) , \quad \alpha_{f_{ii}} = 0 , \\ \alpha_{f_{44}} &= \alpha_{GUT} + O(\alpha_{GUT}^2) , \quad \alpha_p = \frac{15}{7} \alpha_{GUT} + O(\alpha_{GUT}^2) , \end{aligned} \quad (30)$$

where $i = 1, 2, 3$, and the $O(\alpha_{GUT}^2)$ terms are power series in α_{GUT} that can be uniquely computed to any finite order if the β -functions of the unreduced model are known to the corresponding order. The reduced model in which gauge and Yukawa couplings are unified has the β -functions that identically vanish to that order.

In the above model, we found a diagonal solution for the Yukawa couplings, with each family coupled to a different Higgs. However, we may use the fact that mass terms do not influence the β -functions in a certain class of renormalization schemes, and introduce appropriate mass terms that permit us to perform a rotation in the Higgs sector such that only one pair of Higgs doublets, coupled to the third family, remains light and acquires a non-vanishing VEV (in a similar way to what was done by León et al. [25]). Note that the effective coupling of the Higgs doublets to the first family after the rotation is very small avoiding in this way a potential problem with the proton lifetime [31]. Thus, effectively, we have at low energies the MSSM with only one pair of Higgs doublets. Adding soft breaking terms (which are supposed not to influence the β -functions beyond M_{GUT}), we can obtain supersymmetry breaking. The conditions on the soft breaking terms to preserve one-loop finiteness have been given already some time ago [32]. Recently, the same problem at the two-loop level has been addressed [33]. It is an open problem whether there exists a suitable set of conditions on the soft terms for all-loop finiteness. Since the $SU(5)$ symmetry is spontaneously broken below M_{GUT} , the finiteness conditions obviously do not restrict the renormalization property at low energies, and all it remains is a boundary condition on the gauge and Yukawa couplings; these couplings at low energies have to be so chosen that they satisfy (30) at M_{GUT} . So we examine the evolution of the gauge and Yukawa couplings according to their renormalization group equations at two-loops taking into account all the boundary conditions at M_{GUT} . The predictions for m_t for a varying, but common to all particles, supersymmetry breaking threshold are given in Figure 1.

4 The Minimal Asymptotically Free $SU(5)$ Model

Let us consider next the *minimal* $N = 1$ supersymmetric gauge model based on the group $SU(5)$ [5]. Its particle content is then specified and has the following transformation properties under $SU(5)$: three $(\bar{\mathbf{5}} + \mathbf{10})$ - supermultiplets which accommodate three fermion families, one $(\mathbf{5} + \bar{\mathbf{5}})$ to describe the two Higgs supermultiplets appropriate for electroweak symmetry breaking and a $\mathbf{24}$ -supermultiplet required to provide the spontaneous symmetry breaking of $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$.

Since we are neglecting the dimensional parameters and the Yukawa couplings of the first two generations, the superpotential of the model is exactly given by

$$W = \frac{1}{2} g_t \mathbf{10}_3 \mathbf{10}_3 H + g_b \bar{\mathbf{5}}_3 \mathbf{10}_3 \bar{H} + g_\lambda (\mathbf{24})^3 + g_f \bar{H} \mathbf{24} H , \quad (31)$$

where H, \bar{H} are the $\mathbf{5}, \bar{\mathbf{5}}$ - Higgs supermultiplets and we have suppressed the $SU(5)$ indices. According to the notation introduced in Eq. (21), Eqs. (22) become

$$\begin{aligned} \alpha \frac{d\tilde{\alpha}_t}{d\alpha} &= \frac{27}{5} \tilde{\alpha}_t - 3 \tilde{\alpha}_t^2 - \frac{4}{3} \tilde{\alpha}_t \tilde{\alpha}_b - \frac{8}{5} \tilde{\alpha}_t \tilde{\alpha}_f , \\ \alpha \frac{d\tilde{\alpha}_b}{d\alpha} &= \frac{23}{5} \tilde{\alpha}_b - \frac{10}{3} \tilde{\alpha}_b^2 - \tilde{\alpha}_b \tilde{\alpha}_t - \frac{8}{5} \tilde{\alpha}_b \tilde{\alpha}_f , \\ \alpha \frac{d\tilde{\alpha}_\lambda}{d\alpha} &= 9 \tilde{\alpha}_\lambda - \frac{21}{5} \tilde{\alpha}_\lambda^2 - \tilde{\alpha}_\lambda \tilde{\alpha}_f , \\ \alpha \frac{d\tilde{\alpha}_f}{d\alpha} &= \frac{83}{15} \tilde{\alpha}_f - \frac{53}{15} \tilde{\alpha}_f^2 - \tilde{\alpha}_f \tilde{\alpha}_t - \frac{4}{3} \tilde{\alpha}_f \tilde{\alpha}_b - \frac{7}{5} \tilde{\alpha}_f \tilde{\alpha}_\lambda , \end{aligned} \quad (32)$$

in the one-loop approximation. Given the above equations describing the evolution of the four independent couplings (α_i , $i = t, b, \lambda, f$), there exist $2^4 = 16$ non-degenerate solutions corresponding to vanishing ρ 's as well as non-vanishing ones given by Eq. (27). The possibility to predict the top quark mass depends, as in the previous model, on a nontrivial interplay between the vacuum expectation value of the two $SU(2)$ Higgs doublets involved in the model and the known masses of the third generation (m_b , m_τ). It is clear that only the solutions of the form

$$\rho_t , \rho_b \neq 0 \quad (33)$$

can predict the top and bottom quark masses.

There exist exactly four such solutions. The first solution is ruled out since it is inconsistent with Eq. (19), and the second one is ruled out since it does not satisfy the criteria to be asymptotically free. We are left with two asymptotically free solutions, which we label 3 and 4. According to the criteria of section 2, these two solutions give the possibility to obtain partial reductions. To achieve this, we look for solutions [8] of the form Eq. (25) to both 3 and 4.

We present now the computation of some lower order terms within the one-loop approximation for the solutions. For solution 3:

$$\tilde{\alpha}_i = \eta_i + f_i^{(r_\lambda=1)} \tilde{\alpha}_\lambda + f_i^{(r_\lambda=2)} \tilde{\alpha}_\lambda^2 + \dots \quad \text{for } i = t, b, f , \quad (34)$$

where

$$\begin{aligned}
\eta_{t,b,f} &= \frac{2533}{2605}, \frac{1491}{2605}, \frac{560}{521}, \\
f_{t,b,f}^{(r_\lambda=1)} &\simeq 0.018, 0.012, -0.131, \\
f_{t,b,f}^{(r_\lambda=2)} &\simeq 0.005, 0.004, -0.021,
\end{aligned} \tag{35}$$

For the solution 4,

$$\begin{aligned}
\tilde{\alpha}_i &= \eta_i + f_i^{(r_f=1)} \tilde{\alpha}_f + f_i^{(r_\lambda=1)} \tilde{\alpha}_\lambda + f_i^{(r_f=1, r_\lambda=1)} \tilde{\alpha}_f \tilde{\alpha}_\lambda \\
&\quad + f_i^{(r_f=2)} \tilde{\alpha}_f^2 + f_i^{(r_\lambda=2)} \tilde{\alpha}_\lambda^2 \dots \quad \text{for } i = t, b,
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\eta_{t,b} &= \frac{89}{65}, \frac{63}{65}, \quad f_i^{(r_\lambda=1)} = f_i^{(r_\lambda=2)} = 0, \\
f_{t,b}^{(r_f=1)} &\simeq -0.258, -0.213, \quad f_{t,b}^{(r_f=1)} \simeq -0.258, -0.213, \\
f_{t,b}^{(r_f=2)} &\simeq -0.055, -0.050, \quad f_{t,b}^{(r_f=1, r_\lambda=1)} \simeq -0.021, -0.018,
\end{aligned} \tag{37}$$

In the solutions (34) and (36) we have suppressed the contributions from the Yukawa couplings of the first two generations because they are negligibly small.

Presumably, both solutions are related; a numerical analysis on the solutions [8] suggests that the solution 3 is a “boundary” of 4. If it is really so, then there is only one unique reduction solution in the minimal supersymmetric GUT that provides us with the possibility of predicting α_t . Note furthermore that not only α_t but also α_b is predicted in this reduction solution.

Just below the unification scale we would like to obtain the standard $SU(3) \times SU(2) \times U(1)$ model while assuming that all the superpartners are decoupled at the Fermi scale. Then the standard model should be spontaneously broken down to $SU(3) \times U(1)_{\text{em}}$ due to VEV of the two Higgs $SU(2)$ -doublets contained in the $\mathbf{5}, \bar{\mathbf{5}}$ -super-multiplets. One way to obtain the correct low energy theory is to add to the Lagrangian soft supersymmetry breaking terms and to arrange the mass parameters in the superpotential along with the soft breaking terms so that the desired symmetry breaking pattern of the original $SU(5)$ is really the preferred one, all the superpartners are unobservable at present energies, there is no contradiction with proton decay, and so forth. Then, as in the previous model, we

study the evolution of the couplings at two loops respecting all the boundary conditions at M_{GUT} . The predictions for m_t versus M_{SUSY} for the two sets of boundary conditions given above (AFUT3 and AFUT4) together with the corresponding predictions of the FUT model, are given in Figure 1. In a recent study [34], we have considered the proton decay constraint [35] to further reduce the parameter space of the model. It has been found that the model consistent with the non-observation of the proton decay should be very close to AFUT3, implying a better possibility to discriminate between the FUT and AFUT models, as one can see from Figure 4.

5 Asymptotically Non-Free Supersymmetric Pati-Salam Model

In order for the RGI method for the gauge coupling unification to work, the gauge couplings should have the same asymptotic behavior. Note that this common behavior is absent in the standard model with three families. A way to achieve a common asymptotic behavior of all the different gauge couplings is to embed $SU(3)_C \times SU(2)_L \times U(1)_Y$ to some non-abelian gauge group, as it was done in sections 3 and 4. However, in this case still a major role in the GYU is due to the group theoretical aspects of the covering GUT. Here we would like to examine the power of RGI method by considering theories without covering GUTs. We found [9] that the minimal phenomenologically viable model is based on the gauge group of Pati and Salam [1]– $\mathcal{G}_{PS} \equiv SU(4) \times SU(2)_R \times SU(2)_L$. We recall that $N = 1$ supersymmetric models based on this gauge group have been studied with renewed interest because they could in principle be derived from superstring [37].

In our supersymmetric, Gauge-Yukawa unified model based on \mathcal{G}_{PS} [9], three generations of quarks and leptons are accommodated by six chiral supermultiplets, three in $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and three $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, which we denote by $\Psi^{(I)\mu i_R}$ and $\bar{\Psi}_\mu^{(I)i_L}$. (I runs over the three generations, and $\mu, \nu (= 1, 2, 3, 4)$ are the $SU(4)$ indices while $i_R, i_L (= 1, 2)$ stand for the $SU(2)_{L,R}$ indices.) The Higgs supermultiplets in $(\mathbf{4}, \mathbf{2}, \mathbf{1})$, $(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$ and $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ are denoted by $H^{\mu i_R}$, $\bar{H}_{\mu i_R}$ and Σ_ν^μ , respectively. They are responsible for the spontaneous symmetry breaking (SSB) of $SU(4) \times SU(2)_R$ down to $SU(3)_C \times U(1)_Y$. The SSB of

$U(1)_Y \times SU(2)_L$ is then achieved by the nonzero VEV of $h_{i_R i_L}$ which is in $(\mathbf{1}, \mathbf{2}, \mathbf{2})$. In addition to these Higgs supermultiplets, we introduce $G_{\nu i_R i_L}^\mu (\mathbf{15}, \mathbf{2}, \mathbf{2})$, $\phi (\mathbf{1}, \mathbf{1}, \mathbf{1})$ and $\Sigma_\nu'^\mu (\mathbf{15}, \mathbf{1}, \mathbf{1})$. The $G_{\nu i_R i_L}^\mu$ is introduced to realize the $SU(4) \times SU(2)_R \times SU(2)_L$ version of the Georgi-Jarlskog type ansatz [38] for the mass matrix of leptons and quarks while ϕ is supposed to mix with the right-handed neutrino supermultiplets at a high energy scale. With these in mind, we write down the superpotential of the model W , which is the sum of the following superpotentials:

$$\begin{aligned}
W_Y &= \sum_{I,J=1}^3 g_{IJ} \bar{\Psi}_\mu^{(I)i_R} \Psi^{(J)\mu i_L} h_{i_R i_L}, \quad W_{GJ} = g_{GJ} \bar{\Psi}_\mu^{(2)i_R} G_{\nu i_R j_L}^\mu \Psi^{(2)\nu j_L}, \\
W_{NM} &= \sum_{I=1,2,3} g_{I\phi} \epsilon_{i_R j_R} \bar{\Psi}_\mu^{(I)i_R} H^{\mu j_R} \phi, \\
W_{SB} &= g_H \bar{H}_\mu i_R \Sigma_\nu^\mu H^{\nu i_R} + \frac{g_\Sigma}{3} \text{Tr} [\Sigma^3] + \frac{g_{\Sigma'}}{2} \text{Tr} [(\Sigma')^2 \Sigma], \\
W_{TDS} &= \frac{g_G}{2} \epsilon^{i_R j_R} \epsilon^{i_L j_L} \text{Tr} [G_{i_R i_L} \Sigma G_{j_R j_L}], \\
W_M &= m_h h^2 + m_G G^2 + m_\phi \phi^2 + m_H \bar{H} H + m_\Sigma \Sigma^2 + m_{\Sigma'} (\Sigma')^2. \quad (38)
\end{aligned}$$

Although W has the parity, $\phi \rightarrow -\phi$ and $\Sigma' \rightarrow -\Sigma'$, it is not the most general potential, and, by virtue of the non-renormalization theorem, this does not contradict the philosophy of the coupling unification by the RGI method.

We denote the gauge couplings of $SU(4) \times SU(2)_R \times SU(2)_L$ by α_4 , α_{2R} and α_{2L} , respectively. The gauge coupling for $U(1)_Y$, α_1 , normalized in the usual GUT inspired manner, is given by $1/\alpha_1 = 2/5\alpha_4 + 3/5\alpha_{2R}$. In principle, the primary coupling can be any one of the couplings. But it is more convenient to choose a gauge coupling as the primary one because the one-loop β functions for a gauge coupling depends only on its own gauge coupling. For the present model, we use α_{2L} as the primary one. Since the gauge sector for the one-loop β functions is closed, the solutions of the fixed point equations (23) are independent on the Yukawa and Higgs couplings. One easily obtains $\rho_4^{(1)} = 8/9$, $\rho_{2R}^{(1)} = 4/5$, so that the RGI relations (27) at the one-loop level become

$$\tilde{\alpha}_4 = \frac{\alpha_4}{\alpha_{2L}} = \frac{8}{9}, \quad \tilde{\alpha}_1 = \frac{\alpha_1}{\alpha_{2L}} = \frac{5}{6}. \quad (39)$$

The solutions in the Yukawa-Higgs sector strongly depend on the result of the gauge sector. After slightly involved algebraic computations, one finds that most predictive

solutions contain at least three vanishing $\rho_i^{(1)}$'s. Out of these solutions, there are two that exhibit the most predictive power and moreover they satisfy the neutrino mass relation $m_{\nu_\tau} > m_{\nu_\mu}, m_{\nu_e}$. For the first solution we have $\rho_{1\phi}^{(1)} = \rho_{2\phi}^{(1)} = \rho_\Sigma^{(1)} = 0$, while for the second solution, $\rho_{1\phi}^{(1)} = \rho_{2\phi}^{(1)} = \rho_G^{(1)} = 0$, and one finds that for the cases above the power series solutions (27) take the form

$$\begin{aligned}
\tilde{\alpha}_{GJ} &\simeq \begin{cases} 1.67 - 0.05\tilde{\alpha}_{1\phi} + 0.004\tilde{\alpha}_{2\phi} - 0.90\tilde{\alpha}_\Sigma + \dots \\ 2.20 - 0.08\tilde{\alpha}_{2\phi} - 0.05\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_{33} &\simeq \begin{cases} 3.33 + 0.05\tilde{\alpha}_{1\phi} + 0.21\tilde{\alpha}_{2\phi} - 0.02\tilde{\alpha}_\Sigma + \dots \\ 3.40 + 0.05\tilde{\alpha}_{1\phi} - 1.63\tilde{\alpha}_{2\phi} - 0.001\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_{3\phi} &\simeq \begin{cases} 1.43 - 0.58\tilde{\alpha}_{1\phi} - 1.43\tilde{\alpha}_{2\phi} - 0.03\tilde{\alpha}_\Sigma + \dots \\ 0.88 - 0.48\tilde{\alpha}_{1\phi} + 8.83\tilde{\alpha}_{2\phi} + 0.01\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_H &\simeq \begin{cases} 1.08 - 0.03\tilde{\alpha}_{1\phi} + 0.10\tilde{\alpha}_{2\phi} - 0.07\tilde{\alpha}_\Sigma + \dots \\ 2.51 - 0.04\tilde{\alpha}_{1\phi} - 1.68\tilde{\alpha}_{2\phi} - 0.12\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_\Sigma &\simeq \begin{cases} --- \\ 0.40 + 0.01\tilde{\alpha}_{1\phi} - 0.45\tilde{\alpha}_{2\phi} - 0.10\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_{\Sigma'} &\simeq \begin{cases} 4.91 - 0.001\tilde{\alpha}_{1\phi} - 0.03\tilde{\alpha}_{2\phi} - 0.46\tilde{\alpha}_\Sigma + \dots \\ 8.30 + 0.01\tilde{\alpha}_{1\phi} + 1.72\tilde{\alpha}_{2\phi} - 0.36\tilde{\alpha}_G + \dots \end{cases}, \\
\tilde{\alpha}_G &\simeq \begin{cases} 5.59 + 0.02\tilde{\alpha}_{1\phi} - 0.04\tilde{\alpha}_{2\phi} - 1.33\tilde{\alpha}_\Sigma + \dots \\ --- \end{cases}. \tag{40}
\end{aligned}$$

We have assumed that the Yukawa couplings g_{IJ} except for g_{33} vanish. They can be included into RGI relations as small perturbations, but their numerical effects will be rather small.

The number N_H of the Higgses lighter than M_{SUSY} could vary from one to four while the number of those to be taken into account above M_{SUSY} is fixed at four. We have assumed here that $N_H = 1$. The dependence of the top mass on M_{SUSY} in this model is shown in Figure 2.

6 Discussion and Conclusions

As a natural extension of the unification of gauge couplings provided by all GUTs and the model dependent unification of Yukawa couplings, we have introduced in a number of publications the idea of Gauge-Yukawa Unification. GYU is a functional relationship among the gauge and Yukawa couplings provided by some principle. In our studies GYU has been achieved by demanding the principles of reduction of couplings and finiteness. The first principle is based on the existence of RGI relations among couplings which preserve perturbative renormalizability in gauge theories. The second principle is based on the fact that it is possible to find RGI relations among couplings that keep finiteness in perturbation theory, even to all orders. In the previous chapters we have presented the application of these principles in various models as well as the resulting predictions.

The consequence of GYU is that in the lowest order in perturbation theory the gauge and Yukawa couplings above M_{GUT} are related in the form

$$g_i = \kappa_i g_{\text{GUT}} , \quad i = 1, 2, 3, e, \dots, \tau, b, t , \quad (41)$$

where g_i ($i = 1, \dots, t$) stand for the gauge and Yukawa couplings, g_{GUT} is the unified coupling, and we have neglected the Cabibbo-Kobayashi-Maskawa mixing of the quarks. So, Eq. (41) exhibits a boundary condition on the the renormalization group evolution for the effective theory below M_{GUT} , which we assume to be the MSSM.

As we have seen in the previous chapters, there are various supersymmetric GUTs with GYU in the third generation that can predict the bottom and top quark masses in accordance with the experimental data [7, 8, 9]. This means that the top-bottom hierarchy could be explained in these models, exactly in the same way as the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at M_{GUT} [2].

It has been also observed [10] that there exists a relatively wide range of k 's which gives the top-bottom hierarchy of the right order. Of course, the existence of this range is partially related to the infrared behavior of the Yukawa couplings [36]. Therefore, a systematic investigation on the nature of GYU is indispensable to see whether a GYU can make experimentally distinguishable predictions on the top and bottom masses, or

whether the top-bottom hierarchy results mainly from the infrared behavior of the Yukawa couplings. With more precise measurements of the top and bottom masses, we will be able to conclude which case is indeed realized.

We have performed an exhaustive analysis on this problem at the two-loop level [10], and here we would like to present only a few representative results to provide an idea of our complete analysis. We have made the same assumptions as in chapters 3, 4, and 5, namely that below M_{GUT} the evolution of couplings is governed by the MSSM and that there exists a unique threshold M_{SUSY} for all superpartners of the MSSM so that below M_{SUSY} the SM is the correct effective theory, where we include only the logarithmic and two-loop corrections for the RG evolution of couplings. We have neglected all the threshold effects. Note that with a GYU boundary condition alone the value of $\tan \beta$ can not be determined; usually, it is determined in the Higgs sector, which however strongly depends on the supersymmetry breaking terms. In our analysis we avoid this by using the tau mass, along with M_Z , $\alpha_{\text{em}}^{-1}(M_Z)$ and $\sin^2 \theta_W(M_Z)$, as the input. Figure 3 shows the dependence of the top mass on different values of κ_t^2 , when $\kappa_r^2 = \kappa_r^2/\kappa_b^2$ is fixed to 2.0, and the supersymmetry breaking scale $M_{\text{SUSY}} = 500$ GeV.

At this point it is also worth recalling the predictions for m_t of ordinary GUTs, in particular of supersymmetric $SU(5)$ and $SO(10)$. The MSSM with $SU(5)$ Yukawa boundary unification allows m_t to be anywhere in the interval between 100-200 GeV [39] for varying $\tan \beta$, which is now a free parameter. Similarly, the MSSM with $SO(10)$ Yukawa boundary conditions, *i.e.* $t - b - \tau$ Yukawa Unification gives m_t in the interval 160-200 GeV [40].

It is clear that the GYU scenario presented here, provides us with predictive solutions which are consistent with all relevant experimental data. Moreover, it is the most predictive scheme as far as the mass of the top quark is concerned. It is worth noting that the various GYU models can be well distinguished among themselves when more information on the supersymmetry breaking scale is available. A nice demonstration of this point is shown in Figure 4.

Concerning recent related studies, we would like to emphasize that our approach of dealing with asymptotically non-free theories [9] covers work done by other authors [41],

though the underlying idea might be different. Also, interesting RGI relations among the soft breaking parameters above the unification scale have been found [42]. These relations are obtained in close analogy to our approach.

It will be very interesting to find out in the coming years, as the experimental accuracy of m_t increases, if nature is kind enough to verify our conjectured Gauge-Yukawa Unification.

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Figure Captions

Fig. 1. The comparison of the predictions for m_t between the AFUT models and the FUT one. For the FUT model $\tilde{\alpha}_t = 1.6$, $\tilde{\alpha}_b = 1.2$, for AFUT3 $\tilde{\alpha}_t = 0.97$, $\tilde{\alpha}_b = 0.57$, and for AFUT4 $\tilde{\alpha}_t = 1.37$, $\tilde{\alpha}_b = 0.97$.

Fig. 2. The values for m_t predicted by the Pati-Salam model for different M_{SUSY} scales. Only the ones with M_{SUSY} beyond 400 GeV are realistic.

Fig. 3. The dependence of the top mass m_t with k_t^2 , at fixed $M_{SUSY} = 500$ GeV. As we can see, after $k_t^2 \sim 2.0$ the top mass goes to its infrared fixed point value.

Fig. 4. m_t predictions of $SU(5)$ FUT and AFUT3 models, for given M_{SUSY} around 100 and 500 GeV. For the FUT model $\tilde{\alpha}_t = 1.6$, $\tilde{\alpha}_b = 1.2$, and for AFUT3 $\tilde{\alpha}_t = 0.97$, $\tilde{\alpha}_b = 0.57$.







